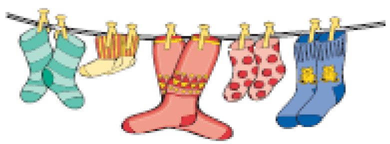
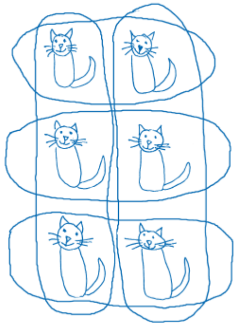
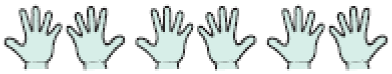
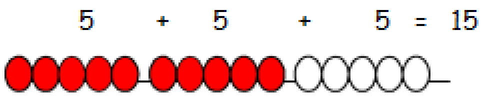
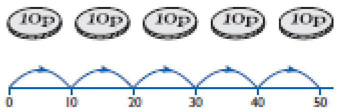
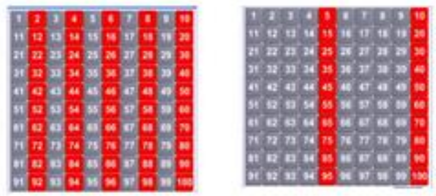
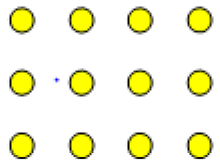
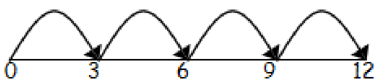
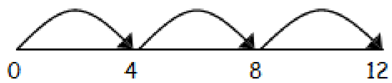
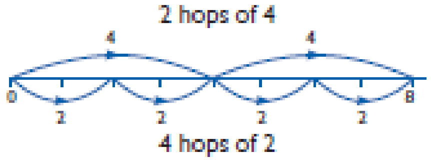
| Progression in Calculations – 6 Steps to Speed and Accuracy | | | |
|---|---|---|---|
| **Addition** | **Subtraction** | **Multiplication** | **Division** |
| **Stage 1:** Recording and developing mental pictures<br>**Stage 2:** Number lines and 100 Squares<br>**Stage 3:** Empty number lines<br>**Stage 4:** Partitioning<br>**Stage 5:** HTU supporting a columnar written method<br>**Stage 6:** Compact column method | **Stage 1:** Recording and developing mental pictures<br>**Stage 2:** Number lines and 100 Squares<br>**Stage 3:** Empty number lines<br>**Stage 4:** Partitioning<br>**Stage 5:** HTU supporting a columnar written method<br>**Stage 6:** Compact column method | **Stage 1:** Recording and developing mental pictures<br>**Stage 2:** Number lines and 100 Squares<br>**Stage 3:** Arrays for multiplication<br>**Stage 4:** Partitioning – The GRID method<br>**Stage 5:** Short multiplication<br>**Stage 6:** Long multiplication | **Stage 1:** Recording and developing mental pictures<br>**Stage 2:** Number lines and 100 Squares<br>**Stage 3:** Arrays for division<br>**Stage 4:** HTU supporting short division<br>**Stage 5:** Short division<br>**Stage 6:** Long division |

# MULTIPLICATION

| Guidance | Examples | Notes |
|---|---|---|
| **Stage 1: Recording and developing mental images**<br>• Children will experience equal groups of objects. They will count in 2s and 10s and begin to count in 5s.<br>• They will experience practical calculation opportunities involving equal sets or groups using a wide variety of equipment, e.g. small world play, role play, counters, cubes etc.<br>• They develop ways of recording<br>• They will see everyday versions of arrays, e.g. egg boxes, baking trays, ice cube trays, wrapping paper etc. They will answer questions such as; 'How many eggs would we need to fill the egg box? How do you know?'<br>• Children will use repeated addition to carry out multiplication using counters/cubes. | **Stage 1**<br><br><br>2 + 2 + 2 + 2 + 2 = 10<br><br>5 + 5 + 5 + 5 + 5 + 5 = 30<br>5 × 6 = 30<br>4 lots of 3 are 12<br>3 lots of 4 are 12<br><br>2 groups of 3 are 6 (3 + 3)<br>3 groups of 2 are 6 (2 + 2 + 2)<br> | Initially recording of calculating should be done by adults to model what children have done in pictures, symbols, numbers and words. Over time there should be an expectation that children will also become involved in the recording process.<br><br>Children should use pictorial representations and may use rings to show e.g. 3 groups of 2 and 2 groups of 3 introducing the commutative law of multiplication |

| | | |
|---|---|---|
| **Stage 2: The bead string, number line and hundred square**<br>• Children continue to use repeated addition to carry out multiplication tasks and represent their counting on a bead string or a number line.<br>• On a bead string, children count out three lots of 5 then count the beads altogether.<br>• On a number line. Children count on in groups of 5. | **Stage 2**<br>3 lots of 5<br><br><br>10p + 10p + 10p + 10p + 10p = 50p<br>10p x 5 = 50p<br>5 hops of 10 | Children begin pattern work on a 100 square to help them begin to recognise multiples and rules of divisibility.<br><br>C[illegible] ay games to reinforce times tables facts and their associated patterns. |
| **Stage 3: Arrays**<br>It is important to be able to visualise multiplication as a rectangular array. This helps children develop their understanding of the commutative law i.e. 3 x 4 = 4 x 3<br><br>The rectangular array allows the total to be found by repeated addition and the link can be made to the 'x' sign and associated vocabulary of 'lots of' 'groups of' etc. | **Stage 3**<br>**3 lots of 4**<br>**3 X 4**<br><br>**4 lots of 3**<br>**4 x 3**<br><br>**4 lots of 3 = 12**<br><br>**3 lots of 4 =12** | The relationship between the array and the number line showing both repeated additions should be demonstrated alongside each other<br><br>For more direct comparison, this could then be demonstrated on a single number line as appropriate. |
| **NOTES: Related calculations/estimates**<br>To utilize further methods, children need to<br>a) know multiplication facts up to 12 x 12<br>b) use place value effectively | E.g. for 47 X 6 they must be able to calculate 40 X 6. They need to recognise the 'root' calculation 4 x 6 = 24<br>and understand that as 40 is ten times greater than 4 the product will also be ten times greater. 40 x 6= 240 | Before carrying out calculations children are encouraged to estimate using rounding e.g.<br>- round a 2 digit number to the nearest 10<br>- round a 3 digit number to the nearest 100 |

## Stage 4: Grid Method

**Two-digit by one-digit products using the grid method (TU x U)**

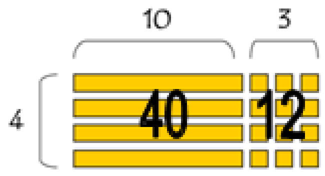
- Children will partition arrays in a variety of helpful ways which are not necessarily the ways in which they will eventually partition them to be in line with formal written methods
- The link between arrays and the grid method should be made clear to children by the use of place value apparatus such as place value counters and Dienes.
- The TU number is partitioned e.g. 13 becomes 10 and 3 and each part of the number is them multiplied by 4.

**Two-digit by two-digit products using the grid method (TU x TU)**

- Children first make an estimate by rounding each number to the nearest ten.
- Having calculated the sections of the grid, children will decide whether to add the rows or columns first as they become more confident with recognising efficient calculations.
- They will choose jottings, informal or formal written methods depending upon which is most appropriate.
- Children should be expected to complete this for TU X TU but not for larger numbers.

## Stage 4

4 x 13

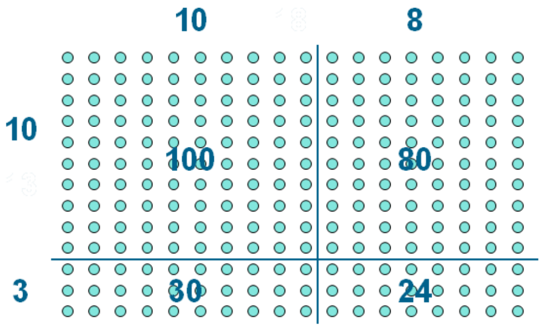



Children should be shown how this model shows 4 x 13 but the calculation steps are 'made easier' by partitioning the 13 into 10 and 3. The use of Dienes emphasises the distributive law.

This then becomes

| x | **10** | **3** |
|---|---|---|
| **4** | 40 | 12 |

40 + 12 = 52




Using pre constructed arrays, children look for ways to split them up using number facts that they are familiar with. Over time this leads to children partitioning two digit numbers into tens and ones, making the link to grid multiplication which is a pre cursor to short and long multiplication.

Children move to the grid method without arrays once they can confidently explain the relationship between the two, even when the array is no longer visible.

## Adding the rows or adding the columns

This should be decided by the child depending on the numbers that are produced through the calculation.

53 x 16

| x | 10 | 6 |
|---|---|---|
| 50 | 500 | 300 |
| 3 | 30 | 18 |

Adding the rows is the most efficient calculation:
500 + 300 = 800
30 + 18 = 48
So 800 + 48 = 848

*Further example:*

38 x 17

| x | 10 | 7 |
|---|---|---|
| 30 | 300 | 210 |
| 8 | 80 | 56 |

Adding the columns:
300 + 80 = 380
210 + 56 = 266
380 + 200 = 580 + 60 = 640 + 6 = 646

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**Stage 5: Short multiplication (TU x U)**

- The first step is to represent the method of recording in a column format, but showing the working. Draw attention to the links with the grid method above.
- Children should describe what they do by referring to the actual values of the digits in the columns. For example, the first step in 38 × 7 is ‘thirty multiplied by seven’, not ‘three times seven’, although the relationship 3 × 7 should be stressed.

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**Stage 5**

Multiply the units first which enables them to move towards the compact method e.g. 38 x 7

30 + 8<br>
X 7<br>
56 7 x 8<br>
210 7 x 30<br>
266

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**Short multiplication (HTU x U)**

- The recording is reduced further, with the carried digits recorded either below the line or at the top of the next column.
- This method is appropriate for multiplying two and three digit numbers by numbers up to 12, which relies on children have recall of their times table facts up to 12.

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342 × 7 becomes

| | | | |
|---|---|---|---|
| | 3 | 4 | 2 |
| × | | | 7 |
| 2 | 3 | 9 | 4 |
| | 2 | 1 | |

Answer: 2394

This example shows the carried digits at the top of the next column

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We have agreed to ‘carry’ the HTU underneath the answer box for a consistent approach.

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**Expanded long multiplication (TU x TU)**

- To ensure understanding of this method, it is important to make direct links to the grid method and may be helpful in the first instance to do both methods side by side to allow children to see the relationship
- There should be an emphasis on making sure that each part of each number is multiplies by each part of the other number

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53 x 16

| x | 10 | 6 |
|---|---|---|
| 50 | 500 | 300 |
| 3 | 30 | 18 |

53<br>
x 16<br>
500 (50 x 10)<br>
300 (50 x 6)<br>
30 (3 x 10)<br>
+ 18 (3 x 6)<br>
848

Staring with the most significant digits makes a more direct link to the grid method.

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<td>Children should be moved towards starting with the column of smallest value as soon as their understanding of the relationship between the methods allows, to move towards long multiplication.<br><br>  53<br>x 16<br>   18 (6 x 3)<br>  300 (6 x 50)<br>   30 (10 x 3)<br>+500 (10 x 50)<br>  848<br><br>Children should be expected to maintain this systematic approach to multiplying numbers, working right to left along the bottom number.  This will ensure that mistakes are not made by ‘missing’ parts, especially when multiplying numbers with more digits.</td>
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<td>Stage 6: Long multiplication<ul><li>Each digit continues to be multiplied by each digit, but the totals are recorded in a more compact form, using ‘carrying’</li><li>Children’s understanding of place value is vital so they recognise when they are multiplying tens, hundreds etc. they record their answer in the correct columns.</li><li>Children should be able to explain each step of the process, initially relating it back to previous methods and experiences.  They should be able to articulate the different stages of this calculation with the true values of the digits they are dealing with.</li></ul></td>
<td>124 × 26 becomes<br><br>    1  2<br>    1  2  4<br>×      2  6<br>    7  4  4<br>2  4  8  0<br>3  2  2  4<br>1  1<br><br>Answer: 3224<br><br>‘Carrying’ can be done above or below the number, but should be consistent as before to avoid mistakes.<br><br>6 x 4 = 24 so record the 4 in the units and carry the 20 (2) into the tens 6 x 20 = 120 + (the carried) 20 = 140 so record the 40 in the tens and carry the 100 (1) into the hundreds column.  6 x 100 = 600 + (the carried) 100 = 700.  Record as 7 in the hundreds.<br><br>20 x 4 = 80 so record this on a new answer row in the correct columns.  20 x 20 = 400. Record the 4 in the hundreds column. 20 x 100 = 2000 so record this appropriately.<br><br>Use column addition to add the two totals together, resulting in 3224.</td>
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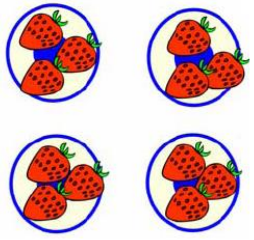
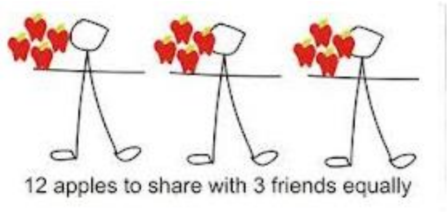
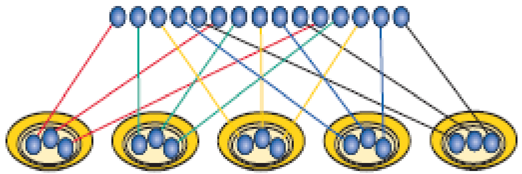
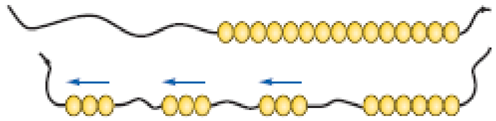
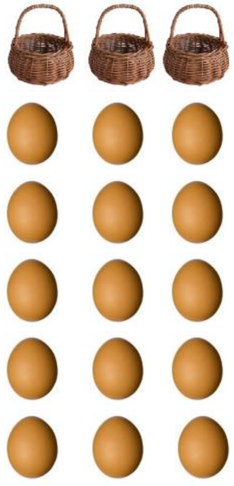
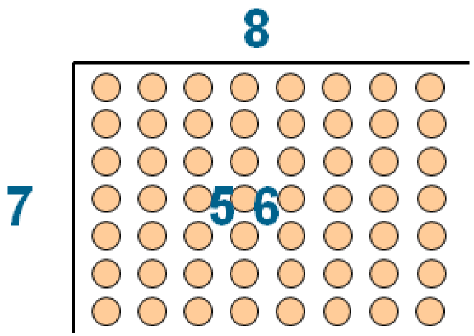
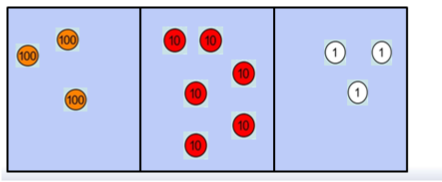
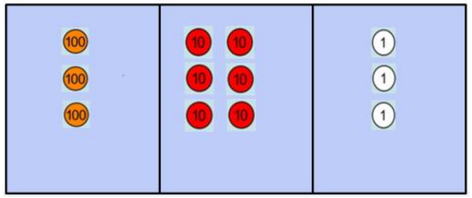
# DIVISION

| Guidance | Examples | Notes |
|---|---|---|
| **Stage 1: Recording and developing mental images**<br>• Children are encouraged, through practical experiences, to develop physical and mental images.<br>• They make recordings of their work as they solve problems where they want to make equal groups of items or sharing objects out equally. | <br><br> | |
| **Sharing and Grouping**<br>• They solve sharing problems by using a 'one for you, one for me' strategy until all of the items have been given out.<br>• Children should find the answer by counting how many eggs **1 basket** has got.<br>• They solve grouping problems by creating groups of the given number.<br>• Children should find the answer by counting out the eggs and finding out how many **groups of 3** there are.<br>• They will begin to use their own jottings to record division | 15 eggs are shared between 5 baskets. How many in each basket?<br>First egg to the first basket, 2nd egg to the second etc<br><br>There are 15 eggs. How many baskets can we make with 3 eggs in?<br> | |

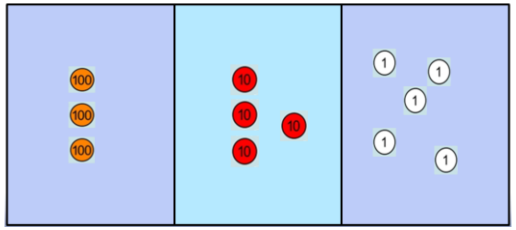
| | | |
|---|---|---|
| **Stage 2: Bead strings, number lines simple multiples**<br>• Using a bead string, children can represent division problems<br>• They count on in equal steps based on adding multiples up to the number to be divided.<br>• When packing eggs into baskets of three they count in threes - **grouping**<br>• If the problem requires 15 eggs to be **shared** between 3 baskets, the multiple of three is obtained each time all three baskets have received an egg. | 15 eggs are placed in baskets, with 3 in each basket. How many baskets are needed?<br><br>Counting on a labelled and then blank number lines.<br>15 ÷ 3 = 5<br><br> | <br>3 eggs once<br>3 eggs twice<br>3 eggs three times<br>3 eggs four times<br>3 eggs five times |
| **Stage 3: Arrays for division**<br>Children construct arrays by grouping the dividend into groups of the divisor. The number of groups made is recorded as the quotient.<br><br>Children then begin to construct the arrays using place value equipment to represent the dividend. | The use of arrays help to reinforce the link between multiplication and<br><br><br>division<br>Divided (56) ÷ divisor (7) = Quotient (8)<br><br>363 ÷ 3<br><br><br>Using the principles of arrays linked to place value becomes: | |



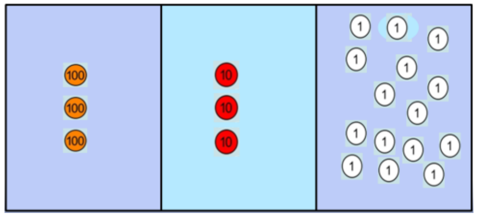
Each part of the number is grouped or shared into the divisor.  Explaining the recording of the division as

$$\begin{array}{r|l} & 1\ 2\ 1 \\ \hline 3 & 3\ 6\ 3 \end{array}$$

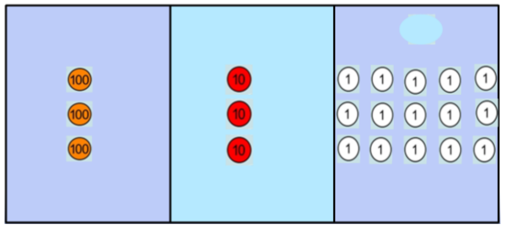
This then becomes more complex when exchange is needed as complete groups of the divisor cannot be made eg.



Then becomes



Which finally becomes



Recorded as

$$\begin{array}{r|l} & 1\ 1\ 5 \\ \hline 3 & 3\ 4\ {}^{1}5 \end{array}$$

This can then be explained in two ways
In one of the three groups, there is one hundred, two tens and one one, making one hundred and twenty one
OR
There is 1 group of three hundreds, 2 groups of three tens and 1 group of three ones making one hundred and twenty one

### Stage 5: Short and Long division

Once children have developed a sound understanding of division, using the manipulatives 'formal written methods' of short and then long division.

### ThHTU x U = Short Division

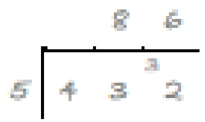
For calculations where numbers with up to 4 digits are divided by a single digit number, children are expected to use short division.

### ThHTU x TU = Long Division

For calculations where numbers of up to 4 digits are divided by a two digit number, children are expected to use long division.

### Short division

432 ÷ 5 becomes

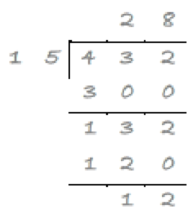


Answer: 86 remainder 2

With short division, children are expected to practise (then practise some more!) and memorise the working method shown above

### Long division

432 ÷ 15 becomes



Answer: 28 remainder 12

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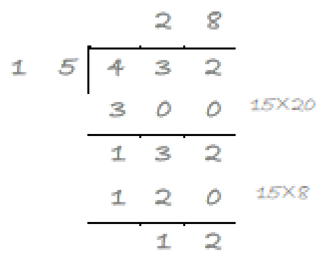
432 ÷ 15 becomes



Answer: 28 $^4/_5$

Children may choose to record the 'chunks' alongside to help them calculate the final answer

And will start to interpret the 'remainder' in the most appropriate way to the context of the question.

By the time children are ready for long division, apparatus may not aid calculating, however they may aid the understanding of the process of long division.

The steps followed can be described as those followed when using PVCs to divide e.g. How many groups of 15 hundreds can we make? None so we exchange the 4 hundreds for 40 tens. How many groups of 15 tens can we make? 2, equivalent to 300. We record the 2 and subtract the 300 that we have 'organised' from the dividend.

We are now left with 132 'ones'. How many groups of 15 can we make with these? 8 and we have 12 left over.