The Aims of The National Curriculum 2014 for Mathematics

The national curriculum for mathematics aims to ensure that all pupils:

- Become **fluent** in the fundamentals of mathematics, including through varied and frequent
- Practice with increasingly complex problems over time, so that pupils develop conceptual understanding and are able to recall and apply their knowledge rapidly and accurately
- **Reason** mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- Can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking Down problems into a series of simpler steps and persevering in seeking solutions.

Additionally, *at Roecliffe*, our curriculum for mathematics aims to ensure that all pupils:

- Develop **positive attitudes** to mathematical learning and progression.
- Have **aspirations** for success in mathematics, deep rooted in high-quality teaching and accessible by all.
- Become **mathematical thinkers**, through hands-on experiences and opportunities to put their mathematics into practice.

C.M.C. = Counting, mental mathematics and written calculations are at the heart of what we teach

Mental and written calculation methods are be taught alongside each other throughout the entirety of the Early Years Foundation Stage, Key Stage 1 and Key Stage 2. Children are actively taught mental mathematics strategies through initiatives such as 'Superhero Maths'. When teachers see that children have 'gaps' in their knowledge, skills or mathematical understanding, developing, they intervene immediately ensuring all children make fast progress throughout the calculation stages.

Progression in Calculations – 6 Steps to Speed and Accuracy			
Addition	**Subtraction**	**Multiplication**	**Division**
Stage 1: Recording and developing mental pictures	**Stage 1:** Recording and developing mental pictures	**Stage 1:** Recording and developing mental pictures	**Stage 1:** Recording and developing mental pictures
Stage 2: Number lines and 100 Squares	**Stage 2:** Number lines and 100 Squares	**Stage 2:** Number lines and 100 Squares	**Stage 2:** Number lines and 100 Squares
Stage 3: Empty number lines	**Stage 3:** Empty number lines	**Stage 3:** Arrays for multiplication	**Stage 3:** Arrays for division
Stage 4: Partitioning	**Stage 4:** Partitioning	**Stage 4:** Partitioning – The GRID method	**Stage 4:** HTU supporting short division
Stage 5: HTU supporting a columnar written method	**Stage 5:** HTU supporting a columnar written method	**Stage 5:** Short multiplication	**Stage 5:** Short division
Stage 6: Compact column method	**Stage 6:** Compact column method	**Stage 6:** Long multiplication	**Stage 6:** Long division

Progress in Calculations - ADDITION

Guidance

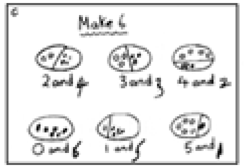
Examples

Stage 1: Recording and developing mental pictures

- Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They experience practical calculation opportunities using a wide variety of equipment, e.g. small world play, role play, counters, cubes etc. They develop ways of recording calculations using pictures, etc.

Stage 1

One and one, two more



makes one, two three."

There are 3 people on the bus. Another person gets on. How many now?

Initially recording of calculating should be done by adults to model what children have done in pictures, symbols, numbers and words. Over time there should be an expectation that children will also become involved in the recording process.

Whilst cameras are an excellent way of keeping a record of what children have done, they are not a substitute for the modelling of different ways of recording calculation procedures.

Stage 2: Progression in the use of a number line

- To help children develop a sound understanding of numbers and to be able to use them confidently in calculation, there needs to progression in their use of number tracks and number lines

The labelled number line

- Children begin to use numbered lines to support their calculations counting on in ones.
- They select the biggest number first i.e. 8 and count on the smaller number in ones.

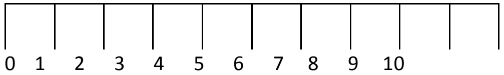
Stage 2

Children should experience a range of representations of number lines, such as the progression listed below.

Number track

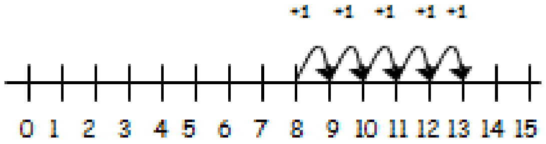
1	2	3	4	5	6	7	8	9	10

Number line, all numbers labelled



- Number line, 5s and 10s labelled
- Number line, 10s labelled
- Number lines, marked but unlabelled

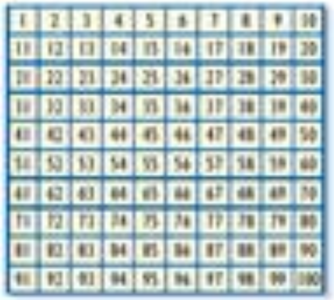
8 + 5 = 13



Additional 'number lines' - The bead string and hundred square

A hundred square is an efficient visual resource to support adding on in ones and tens and is an extension to the number track that children have experienced previously.

8 + 2 = 10



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Along with the number line, bead strings can be used to illustrate addition.

Stage 3: The empty number line as a representation of a mental strategy

NB It is important to note that the empty number line is intended to be a representation of a mental method, not a written algorithm (method). Therefore the order and size (physical and numerical) of the jumps should be expected to vary from one calculation to the next.

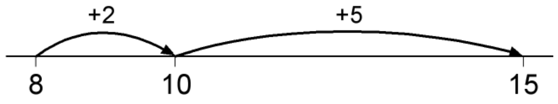
- The mental methods that lead to column addition generally involve partitioning.
- <u>Children need to be able to partition numbers in ways other than into tens and ones to help them make multiples of ten by adding in steps.</u>
- The empty number line helps to record the steps on the way to calculating the total.

Stage 3

Steps in addition can be recorded on a number line. The steps often bridge through a multiple of 10.

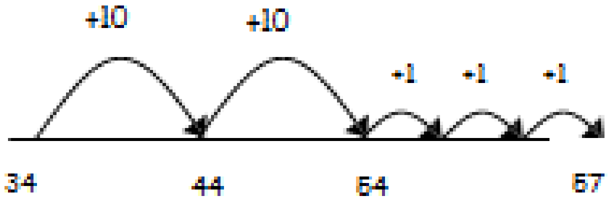
8 + 7 = 15

Seven is partitioned into 2 and 5; 2 creating a number bond to 10 with the 8 and then the 5 is added to the 10.



First counting on in tens and ones.

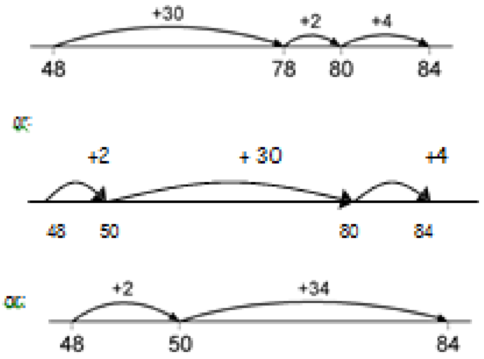
34 + 23 = 57



This develops in efficiency, alongside children's confidence with place value.

Counting on in multiples of 10.

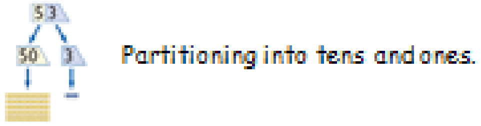
48 + 36 = 84



These examples show how children should be taught to use jumps of different sizes, and completed in an order that is most helpful depending on the numbers they are calculating with. **This reinforces that this is a visual representation of a mental method and not a written alogrithm.**

Stage 4: Partitioning into tens and ones to lead to a formal written method

- The next stage is to record mental methods using partitioning into tens and ones separately.



- Add the tens and then the ones to form partial sums and then add these partial sums.
- Partitioning both numbers into tens and ones mirrors the column method where ones are placed under ones and tens under tens. This also links to mental methods.
- This method can be extended for TU + HTU and HTU + HTU and beyond; as well as cater for the addition of decimal numbers.

Stage 4

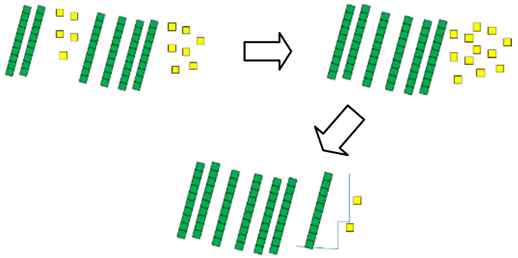
Children should use a range of practical apparatus (place value cards, straws, Dienes apparatus, place value counters) to complete TU + TU.

They partition the number into tens and ones before adding the numbers together, finding the total.

There should be progression through this selection of apparatus. Once using abstract representations teachers will start with straws, bundled into 10s and singularly. Children see 10 straws making one bundle and can be involved in bundling and unbundling.

Once children are able to use these with understanding, they will progress to the use of place value cards and place value counters which are a further abstraction of the concept of number. Money

15 + 47

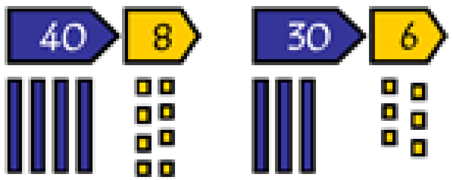
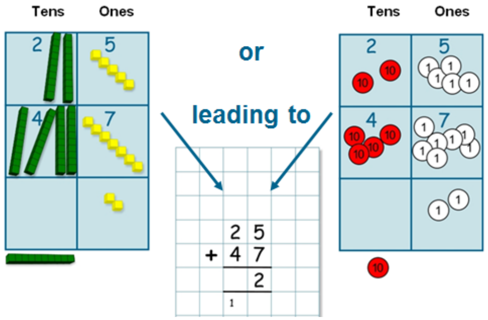


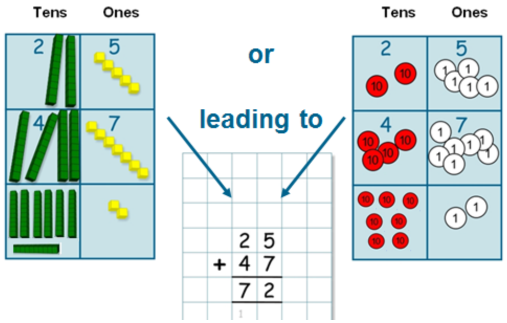
Children may make these jottings to support their calculation.

47 + 76

40 + 70 = 110 **or** 7 + 6 = 13

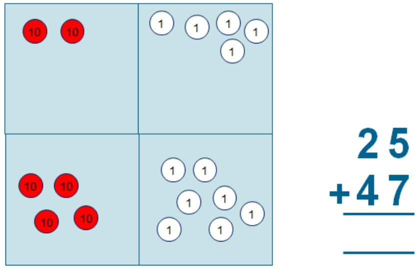
7 + 6 =13 40 + 70 = 110

	should also be used (1ps, 10ps and £1) as place value equipment to help children develop their understanding of different representations. Progress through these manipulatives should be guided by understanding not age or year group. 48 + 36  40 + 30 = 70 8 + 6 = 14 70 + 14 = 84 Cuisenaire can also be used to support this step, especially when crossing the tens barrier with ones. When this occurs, children should use the term 'exchange' to describe converting ten ones into one ten.	110+ 13 = 123 110 + 13 = 123 **or** 47 + 70 = 117 117 + 6 = 123
Stage 5 – Using Dienes/place value counters alongside columnar written method • To ensure the statutory final written method is grounded in understanding, this stage connects the practical equipment to the formal written method using a similar and transferrable layout. • Children first experience the practical version of column addition and when confident in explaining this, including exchanging when crossing the tens barrier with ones, they record the written method alongside. • Ideally children will experience this stage with a variety of practical equipment to make sure their understanding is embedded and transferrable between representations. • Children may learn more from experiencing the inefficiency of not starting with column with least significant value rather than being 'told' where to start.	**Stage 5 – Hundreds, Tens and Ones / PV Counters** **It may be appropriate to teach children the process with numbers that they would be expected to calculate mentally or with jottings. This is to aid with the practicalities of the use of such equipment. However this should be the exception rather than the rule so children see a clear purpose for learning a new method for calculating.** In this example 25 + 47 =  	Represented in place value columns and rows. Starting adding with the 'least significant digit' When the tens barriers is crossed in the 'ones' exchange then takes place.

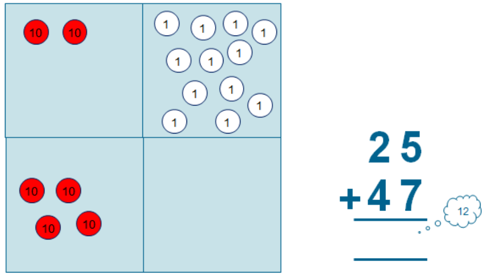


Whilst these images show the total existing alongside the two numbers being added, it may be more representative to ‘drag’ the manipulatives down to the totals box, leaving the written numbers as a reminder of what was originally there.

Another way of representing this is like this



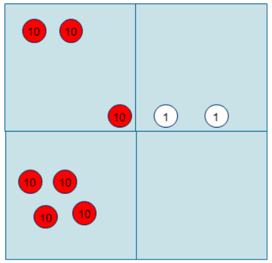
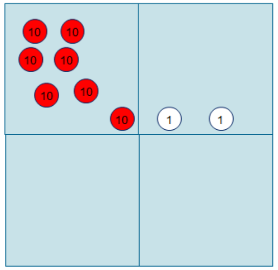
Where the bottom value is combined with the top value



Because of the exchange we can know see that this ten belongs in the tens column and is carried there to be included in the total of that column.

The tens are then added together 20 + 40 + 10 = 70, recorded as 7 in the tens column.

This method aligns with the approaches used in some intervention programmes and involves less movement of equipment however does not match as closely to the columnar abstract representation as the suggestion above.

	 $\begin{array}{r} 25 \\ +47 \\ \hline 2 \\ \hline {}_{1} \end{array}$  $\begin{array}{r} 25 \\ +47 \\ \hline 72 \\ \hline {}_{1} \end{array}$	
Stage 6: Compact column method • In this method, recording is reduced further. Carried digits are recorded, using the words ‘carry ten’ or ‘carry one hundred’ etc., according to the value of the digit. • Later the method is extended when adding more complex combinations such as three two-digit numbers, two three-digit numbers, and problems involving several numbers of different sizes.	**Stage 6** $\begin{array}{r} 258 \\ +\ 87 \\ \hline 345 \\ \hline {}_{1\,1} \end{array}$ $\quad$ $\begin{array}{r} 366 \\ +458 \\ \hline 824 \\ \hline {}_{11} \end{array}$ **Column addition remains efficient when used with larger whole numbers and once learned, is quick and reliable.** Th H T U $\begin{array}{r} 3674 \\ +\ 2507 \\ \hline 6181 \\ \hline {}_{1\ \ 1} \end{array}$	

Progress in Calculations - SUBTRACTION

Guidance

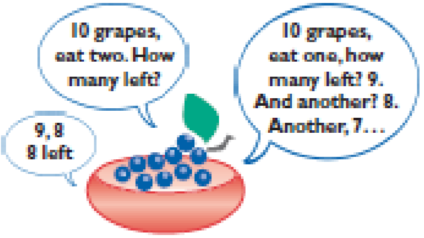
Examples

Stage 1: Recording and developing mental pictures

Children are encouraged to develop a mental picture of the calculation in their heads. They experience practical activities using a variety of equipment and develop ways to record their findings including models and pictures.

- The ‘difference between’ is introduced through practical situations and images.

Stage 1



There are four children in the role play corner. One leaves. How many are left?

Initially recording of calculating should be done by adults to model what children have done in pictures, symbols, numbers and words. Over time there should be an expectation that children will also become involved in the recording process.

Whilst cameras are an excellent way of keeping a record of what children have done, they are not a substitute for the modelling of different ways of recording calculation procedures.

Stage 2: Progression in the use of a number line

- Finding out how many items are left after some have been ‘taken away’ is initially supported with a number track followed by labelled, unlabelled and finally empty number lines, as with addition.

The labelled number line

- The labelled number line, linked with previous learning experiences, is used to support calculations where the result is less objects (i.e. taking away) by counting back.

Difference between

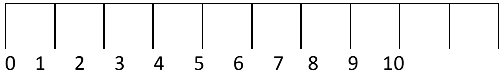
- The number line should also be used to make comparisons between numbers, to show that 6 – 3 means the ‘**difference in value** between 6 and 3’ or the ‘difference between3 and 6’ and how many jumps they are apart.

Stage 2

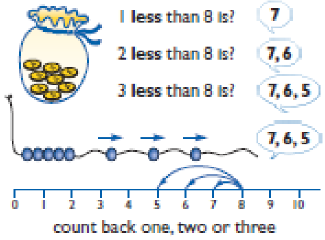
Children should experience a range of representations of number lines, such as the progression listed below. Number track

1	2	3	4	5	6	7	8	9	10

Number line, all numbers labelled

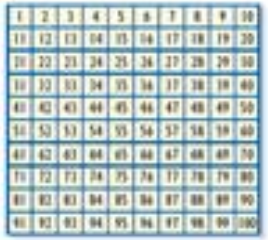


- Number line, 5s and 10s labelled
- Number line, 10s labelled
- Number lines, marked but unlabelled

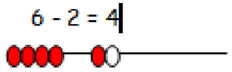


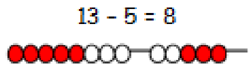
Additional ‘number lines’ - The bead string and hundred square

A hundred square is an efficient visual resource to support counting on and back in ones and tens and is an extension of the number track



Bead strings can be used to illustrate subtraction. 6 beads are counted and then the 2 beads taken away to leave 4.





Stage 3: The empty number line as a representation of a mental strategy

NB It is important to note that the empty number line is intended to be a representation of a mental method, not a written algorithm (method). Therefore the order and size (physical and numerical) of the jumps should be expected to vary from one calculation to the next.

Finding an answer by COUNTING BACK

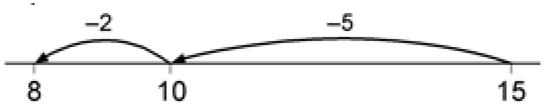
- **Counting back** is a useful strategy when the context of the problem results in there being less e.g. Bill has 15 sweets and gives 7 to his friend Jack, how many does he have left? As in addition, children need to be able to partition numbers e.g. the 7 is partitioned into 5 and 2 to enable counting back to 10.
- The empty number line helps to record or explain the steps in mental subtraction.
- A calculation like 74 – 27 can be recorded by counting back 27 from 74 to reach 47. The empty number line is a useful way of modelling processes such as bridging through a multiple of ten.

Stage 3

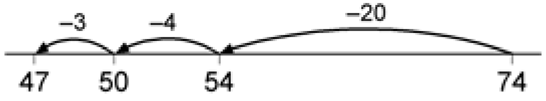
Steps in subtraction can be recorded on a number line. The steps often bridge through a multiple of 10.

15 – 7 = 8

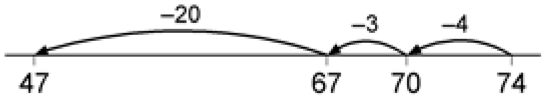
The seven is partitioned into 5 (to allow count back to 10) and two.



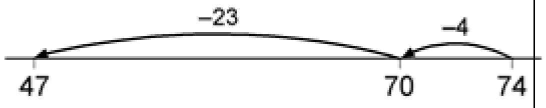
74 – 27 = 47 worked by counting back:



The steps may be recorded in a different order:



or combined



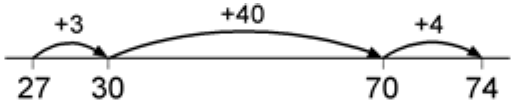
These examples show how children should be taught to use jumps of different sizes, and completed in an order that is most helpful depending on the numbers they are calculating with. **This reinforces that this is a visual representation of a mental method and not a written alogrithm.**

Stage 4: Using empty number lines / COUNTING ON

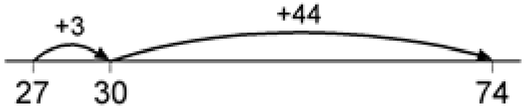
- The steps can also be recorded by **counting on** from the smaller to the larger number to find the difference, for example by counting up from 27 to 74 in steps totalling 47 (shopkeeper's method). This is a useful method when the context asks for comparisons e.g. how much longer, how much smaller; for example: Jill has knitted 27cm of her scarf, Alex has knitted 74cm. How much longer is Alex's scarf?
- After practice of both, examples like this will illustrate how children might choose when it is appropriate to count on or back. This also helps to reinforce addition and subtraction as inverses and the links between known number facts.

Stage 4

74 – 27 =



The 'jumps' should be added, either mentally or with jottings according to confidence, beginning with the largest number e.g. 40 + 4 + 3.



Stage 5: Practical equipment using exchange to 'take away'

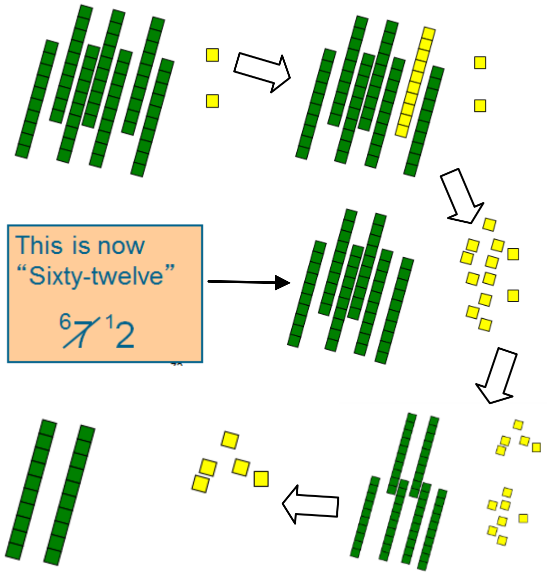
- Children use practical apparatus to take away the smaller number from the larger. This should be used to model exchanging as in the example.
- Children's place value knowledge should be good enough to understand that the change still represents the original starting number and is just a different way of partitioning it.

Stage 5

There should be progression through this selection of apparatus.
Once using abstract representations teachers will start with straws, bundled into 10s and singularly. Children see 10 straws making one bundle and can be involved in bundling and unbundling.
This then progresses to the use of Dienes (or similar) where 10s are clearly ten ones but cannot be separated in the same way.
Once children are able to use these with understanding, they will progress to the use of place value cards and place value counters which are a further abstraction of the concept of number. Money should also be used (1ps, 10ps and £1) as place value equipment to help children develop their understanding of a range of representations.

Progress through these manipulatives should be guided by understanding not age or year group.

72 – 47



72 – 47 = 25

This stage should also be represented using place value counters, going through the same process as the Dienes example.

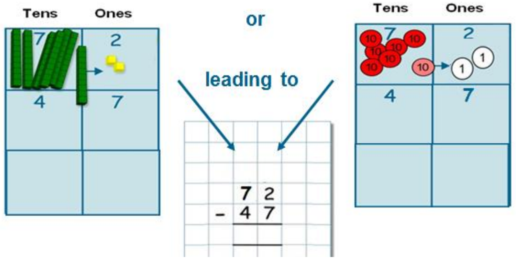
Because of the cumbersome nature of 'exchanges' in this form, examples that children are expected to do with the practical equipment should be limited to HTU – HTU with one exchange in each calculation

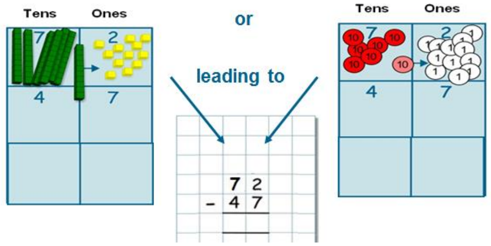
Stage 5: Making the link between the practical and columnar subtraction

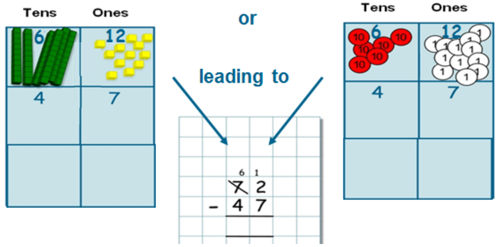
- To ensure the statutory final written method is grounded in understanding, this stage connects the practical equipment to the formal written method using a similar and transferrable layout.
- Children first experience the practical version of column subtraction and when confident in explaining this, including exchanging when ‘not having enough to subtract from’, they record the written method alongside.
- Ideally children will experience this stage with a variety of practical equipment to make sure their understanding is embedded and transferrable between representations.
  - Children may learn more from experiencing the inefficiency of not starting with column with least significant value than being ‘told’ where to start.

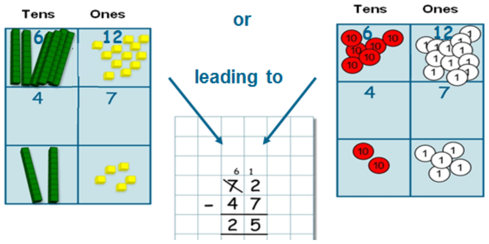
Stage 5

72 - 47

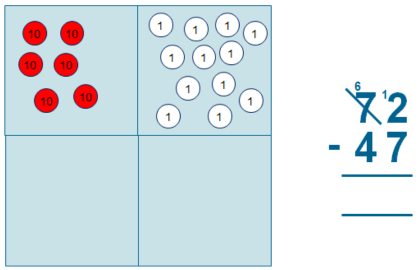
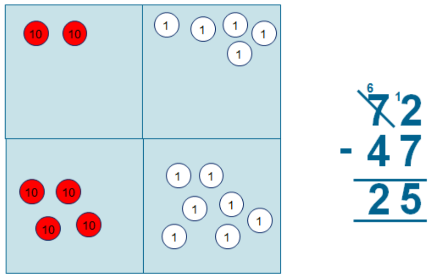
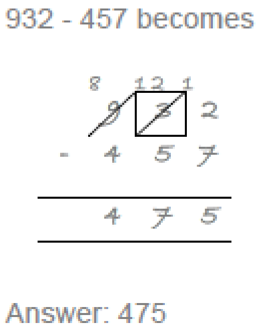
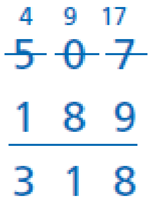








Whilst the images here show the total existing alongside the original number, it is suggested that the 47 would be ‘removed’ from the original set, before ‘dragging’ what is left down to the totals box. This would more closely represent the written algorithm.

	Another way of representing this is;  Where the subtracted amount is removed  The answer is represented by what is remaining in the top row	This method aligns with the approaches used in some interventions and involves less movement of equipment however does not match as closely to the columnar abstract representation as the suggestion above.
Stage 6: Compact method - Finally children complete the compact columnar subtraction as the most efficient form. - Once children are confident with HTU – HTU, this should be extended to four digit subtract four digit calculations.	**Stage 6** 563 – 246 = 317   932 - 457 becomes Answer: 475	 Children may find it more helpful to present their exchanges like this to keep the numbers clear.